

Grade Level: 10-11  
Discipline: Mathematics  
Algebra 2: Functions  
Heterogeneous Classroom

**Purpose:** We will begin this unit with a “function walk”. It will take place in the gymnasium. I will set up lawn chair webbing as the x, y coordinate plane. Students will be given an x card with an integer value on it and then the class will be given an equation. The students will be asked to move to their solution. They will sit and sketch the graph of the class. Next, they will determine the domain of the equation by standing up and while staying on the same x coordinate they will move off of their y coordinate and walk toward the x axis. They will squish” the graph. As a group, we will examine our new graph and determine domain. We will do the same in the y direction to determine range. From this introductory activity students will generate a list of 3 types of equations with many examples. They will use these 3 examples throughout the unit as a comparative tool.

Comparison and analysis is necessary in this unit. The method of comparison will be different for each student. Some will look at graphs and “see a relationship”. Some students need to “see the numbers “to determine a relationship. Differentiation will occur in their method of learning to recognize relationships between functions. It will also be part of the assessment of this unit. All students will be required to answer the first two tiers of questions from this unit to receive a passing grade. The third tier is a goal.

**Type of management system:** During this unit there will be whole group instruction, small group work and independent project work.

**Scheduling Time:** Class will begin with a quiz on the previous day’s topic. Next, there is whole group instruction, followed by practice. For this unit practice consists of small group work Homework, or as I like to refer to it as “supplemental practice” will generally be assigned nightly. This is determined on a day to day basis, as needed. There is an individual project required for this unit. Students may work on their project, throughout the unit, when required assignments are completed. One day will be scheduled for a review and discussion of the projects. Students will be required to design and or make a function machine.

**Assessment Types and Procedures:** The goal of any teacher is that all students will possess the necessary skills before moving on in a subject. This is essential in a math class. Pre-assessment is not necessary, this course is sequential and I already know the skill level of my students. Tiered questions will be useful in determining comprehension. An understanding of the first two tiered questions is a necessity. Attainment of the third is a goal.

There will be a variety of assessments throughout this unit. Students will be required to keep a “tool box” of examples of different types of functions, which will be graded using

a check off sheet. It is either there or not. There will be daily quizzes and a culminating test. There will be homework assignments and daily in class work that will be assessed based on a questioning sheet. There will also be a project graded on a rubric.

**1. Unit Content**

**a. Domain**

- 1. of linear functions**
- 2. of quadratic functions**
- 3. of cubic functions**
- 4. of square root functions**

**b. Range**

- 1. of linear functions**
- 2. of quadratic functions**
- 3. of cubic functions**
- 4. of square root functions**

**c. Relations vs. Functions**

- 1. Mapping diagrams**
- 2. Ordered pairs**
- 3. Graphing**
- 4. Equations**

**d. Function Notation**

- 1. Replacing  $y$**
- 2.  $f(x)$**
- 3. inverse of  $f(x)$**
- 4. Composition:  $f(g(x))$**

**e. Function Operations**

- 1. Addition**
- 2. Subtraction**
- 3. Multiplication**
- 4. Division**
- 5. Composition**

**f. Inverse Functions**

- 1. Vertical line test**
- 2. Horizontal line test**

**g. Recursive Functions**

- 1. Evaluating recursive functions**
- 3. as a linear model**
- 4. as a quadratic model**

- h. Translations**
  - 1. of linear functions
  - 2. of quadratic functions
  - 3. of cubic functions
  - 4. of square root functions
  
- I. Step functions**
  - 1. Evaluate
  - 2. Graph
  
- J. Compound Functions**
  - 3. Evaluate
  - 4. Graph

MLR : G 1-4  
H 3-4  
K 1-2

## Essential Questions

- A. How is a function like a machine?
- B. If all functions are relations why aren't all relations functions?

## Essential Unit Questions

### Tiered Questions

1. **What is the domain for linear, quadratic, and exponential equations?**
  - 1.1 Given a graph of each type of equation: linear, quadratic and exponential, list the domain using set notation.
  - 1.2 Write 3 equations whose domains are:
    1.  $x: x \in \mathbb{R}$
    2.  $x: x \in \mathbb{R} \ \& \ x \neq 1$
    3.  $x: x \in \mathbb{R} \ \& \ x \geq 0$
  - 1.3 Describe how limiting the domain of a function determines the value of that function and apply that knowledge to justify limiting the domain of a square root function.
2. **What is the range for linear, quadratic, and exponential equations?**
  - 2.1 Given a graph of each type of equation: linear, quadratic and exponential, list the range using set notation.
  - 2.2 Write 3 equations from the following categories: linear, quadratic and square root, whose range is:
    1.  $x: x \in \mathbb{R}$
    2.  $x: x \in \mathbb{R} \ \& \ x \neq 1$
    3.  $x: x \in \mathbb{R} \ \& \ x \geq -1$
  - 2.3 Using the graphing calculator and it's table, determine the asymptotes of the given equations and hypothesize a general rule to determine the range of an asymptotic equation. Write your determination in your "tool box."
3. **What are the differences between a relation and a function?**
  - 3.1 Using a mapping diagram, an ordered pairs a graph and an equation explain verbally how to recognize a function. Specifically, explaining the use of the vertical line test.
  - 3.2 Analyze a series of given graphs to determine if they represent a relation or a function. Label each function and include these in your "tool box".

3.3 Create 3 equations that are relations and functions. Next create 3 equations that represent relations only but are not functions. You may use your graphing calculator to experiment with various equations. Include these 3 examples in your toolbox.

#### **4. Using function notation, how are function operations performed?**

4.1 Define the five function operations: addition, subtraction, multiplication, division and composition using function notation. Write examples of each.

4.2 Calculate the composition of  $f(g(x))$  and  $g(f(x))$  for a combination of quadratic and linear equations from the given list. Choose 2 and solve.

4.3 Given a function  $h(x) = x^2 + 2$ , create 2 functions  $f$  and  $g$  such that  $h(x) = f(g(x))$ . And then calculate  $g(f(x))$ .

#### **5. What is the inverse of a function?**

5.1 Define the inverse of a function and demonstrate how to determine the inverse of a linear, quadratic and square root function. Explain verbally to the teacher.

5.2 Given the inverse of a function in each category determine the equation of  $f(x)$ .

5.3 Is the inverse of a function, a function? If yes, explain why. If not, use function notation and function operations to combine non-function inverses and create a function. Whew!

#### **6. What is a recursive function and how are they evaluated?**

6.1 Define a recursive function and its domain and explain why the factorial function meets this criteria.

6.2 Evaluate the given recursive functions. Determine the first and second Differences and classify them as linear, quadratic or neither.

6.3 Find the first 9 values of the Fibonacci Sequence and analyze whether there is a linear, quadratic or cubic model to describe this function, explain why or why not. Now hypothesize a real life recursive situation that could be modeled by a linear model.

#### **7. How is function notation used to translate graphs?**

7.1 Given a graph from each of the following categories: linear, quadratic, square root and cubic state the five possible graphic shifts that could be applied and sketch an example of each.

7.2 Given an algebraic model that describes the population growth in the United States between 1980 and 2005 translate this model 10 years into the future and estimate the population in 2015.

7.3 Given two quadratic equations, translate the equations to construct a cubic model. You may use function operations and you may limit the domain.

## **8. What is a step function and in what situations would it be an appropriate model?**

8.1 Define a step function and determine the domain and the range given a graph.

8.2 Given a step function graph create a scenario that could be used to describe the model.

8.3 Given two step functions, justify why one over the other would better model the given situation.

## **9. What is a compound function and how is one evaluated?**

9.1 Define a compound function and sketch an example of a quadratic / linear function. State its domain and range.

9.2 Write an absolute value function as compound function. State its domain and range.

9.3 Given the graph of an elaborate compound function ( a combination of 3 overlapping functions). Determine the system of equations used to describe this graph and justify your choice of domain by demonstrating that they intersect appropriately the domain for each.

## **10. How is a function like a machine?**

10.2 Define a function and apply that definition to a candy machine. How is a function like a candy machine. How is a function not like a candy machine? As a group we will brainstorm a list of ideas and create a list.

10.2 Using the list above design a function machine. Either create an actual machine or sketch a blueprint. Include a manual that describes how your machine works. Define the domain and the range of your machine.

10.3 Present your machine to the class and justify how it meets the criteria of a function. Be prepared to compare and contrast your machine with the teacher's non-function machine. You may examine the teacher's machine prior to the lesson.

### Rubric for a Function Machine

Criteria	1	2	3	4
<p><b>Knowledge of what a function is and how that definition applies to the machine.</b></p>	<p><b>Student does not understand the definition of a function</b></p>	<p><b>Student states the correct definition but does not apply the definition correctly to their machine.</b></p>	<p><b>The student understands the correct definition but their machine is flawed.</b></p>	<p><b>The student understands the correct definition and has created true function machine. Move over Henry Ford!</b></p>
<p><b>Compare and contrast their function machine with the teacher's non-function machine.</b></p>	<p><b>Student can not determine the difference between the two products.</b></p>	<p><b>Student can explain why their product is a function but does not recognize why the teacher's is not a function.</b></p>	<p><b>Student states the correct conclusion but does not support their opinion with accurate observations.</b></p>	<p><b>They see it! They tell everyone! They use accurate observations and definitions.</b></p>
<p><b>Product: a completed design or actual machine with an accompanying manual on it's use</b></p>	<p><b>Poorly constructed machine. Manual is missing</b></p>	<p><b>Machine is well constructed. Manual is missing.</b></p>	<p><b>Machine has no major flaws. Manual has no major flaws</b></p>	<p><b>Ready to sell. Let's get this baby to market!</b></p>